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EQUAÇÕES DE MOVIMENTO – FORÇA DEPENDENTE DO TEMPO

$\vec{F}(t)$. $\quad 2^a$ L.N. : $m\dfrac{d\vec{v}}{dt} = \vec{F}(t)$.

$$\int_{\vec{v}(0)}^{\vec{v}(t)} d\vec{v} = \int_0^t \frac{1}{m}\vec{F}(t)dt \Rightarrow \boxed{\vec{v}(t) = \vec{v}(0) + \frac{1}{m}\int_0^t \vec{F}(t)\,dt}$$

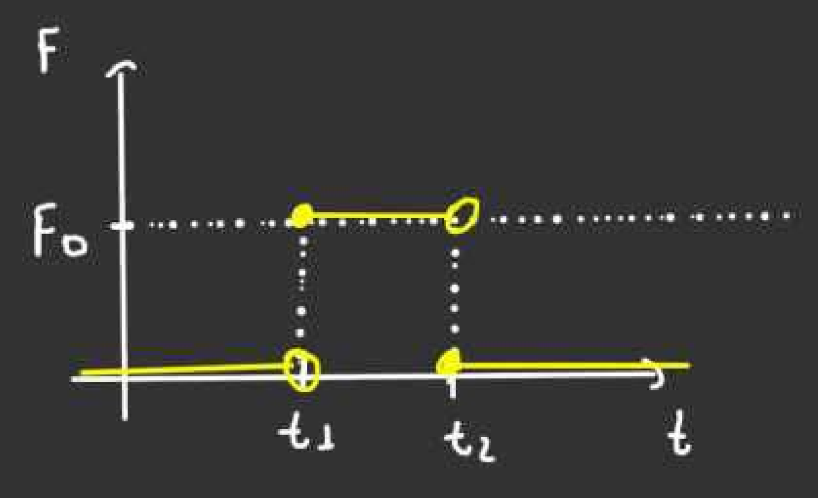
$$\boxed{\Delta\vec{p} = \int \vec{F}(t)dt} \rightarrow \text{Teorema do impulso.}$$

$$\frac{d\vec{r}}{dt} = \vec{v}(0) + \frac{1}{m}\int_0^t \vec{F}(t)dt$$

$$\int_{\vec{r}(0)}^{\vec{r}(t)} d\vec{r} = \int_0^t \left[\vec{v}(0) + \frac{1}{m}\int_0^t \vec{F}(t')dt'\right]dt$$

$$\boxed{\vec{r}(t) = \vec{r}(0) + \vec{v}(0)t + \frac{1}{m}\int_0^t\left(\int_0^t \vec{F}(t')dt'\right)dt}$$

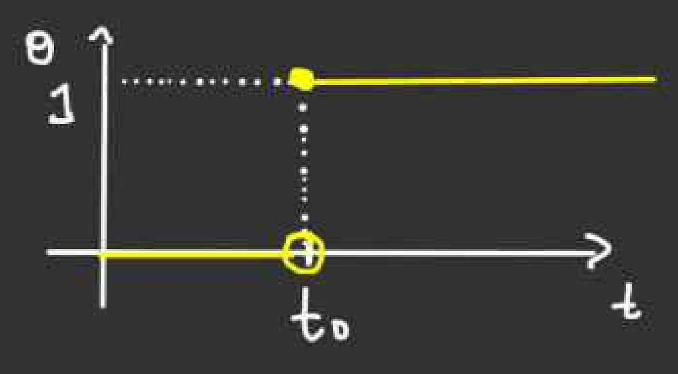
Exemplo: Força impulsiva



$t_2 > t_1$

$$F(t) = F_0\left[\Theta(t-t_1) - \Theta(t-t_2)\right]$$

$$\Theta(t-t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t \geq t_0 \end{cases}$$



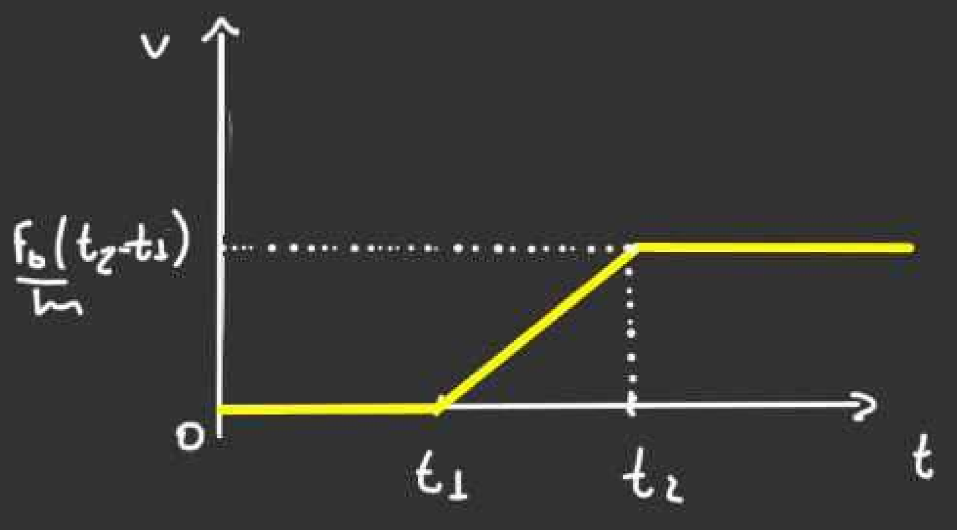
Assumir $v(0) = 0$

$$v(t) = \frac{1}{m}\int_0^t F(t)dt.$$

$$= \frac{F_0}{m}\int_0^t [\Theta(t-t_1) - \Theta(t-t_2)]dt$$

$$= \frac{F_0}{m}\begin{cases} \int_0^t 0\,dt, & t < t_1 \\ \int_0^{t_1} 0\,dt + \int_{t_1}^t dt, & t_1 \leq t < t_2 \\ \int_0^{t_1} 0\,dt + \int_{t_1}^{t_2} dt + \int_{t_2}^t 0\,dt, & t \geq t_2 \end{cases}$$

$$v(t) = \frac{F_0}{m}\begin{cases} 0, & t < t_1, \\ t - t_1, & t_1 \leq t < t_2, \\ t_2 - t_1, & t \geq t_2. \end{cases}$$



$x(t) = ?$

$$\frac{dx}{dt} = v(t) \Rightarrow \int_{x(0)}^{x(t)} dx = \int_0^t v\,dt \Rightarrow x(t) = x(0) + \int_0^t v(t)dt$$

$$\int_0^t v(t)dt = \begin{cases} \int_0^t 0\,dt, & t < t_1 \\ \int_0^{t_1} 0\,dt + \int_{t_1}^t \frac{F_0}{m}(t-t_1)dt, & t_1 \leq t < t. \\ \int_0^{t_1} 0\,dt + \int_{t_1}^{t_2} \frac{F_0}{m}(t-t_1)dt + \int_{t_2}^t \frac{F_0}{m}(t_2-t_1)dt \end{cases}$$

$$= \begin{cases} 0, & t < t_1 \\ \frac{F_0}{m}\int_0^{t-t_1} u\,du = \frac{F_0}{m}\frac{(t-t_1)^2}{2}, & t_1 \leq t < t_2 \\ \frac{F_0}{m}\int_0^{t_2-t_1} u\,du + \frac{F_0}{m}(t_2-t_1)(t-t_2), & t \geq t_2 \end{cases}$$

$$= \begin{cases} 0, & t < t_1 \\ \frac{F_0}{2m}(t-t_1)^2, & t_1 \leq t < t_2 \\ \frac{F_0}{2m}(t_2-t_1)^2 + \frac{F_0}{m}(t_2-t_1)(t-t_2) = \frac{F_0}{2m}\left[t_1^2 - t_2^2 + 2t(t_2-t_1)\right] \end{cases}$$

$$x(t) - x(0) = \frac{F_0}{2m}\begin{cases} 0, & t < t_1 \\ (t-t_1)^2, & t_1 \leq t < t_2 \\ t_1^2 - t_2^2 + 2t(t_2-t_1), & t \geq t_2 \end{cases}$$