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QUAL A VELOCIDADE TERMINAL DE UMA GOTA DE CHUVA?

FENOMENOLOGIA DO ARRASTO

Arrasto sobre um corpo: $F_{ar} \propto R^a \rho^b \eta^c v^d$,

$$[M]\frac{[L]}{[T]^2} = [L]^a \frac{[M]^b}{[L]^{3b}} \cdot \frac{[M]^c}{[L]^c[T]^c} \cdot \frac{[L]^d}{[T]^d} = \frac{[M]^{b+c}\,[L]^{a+d-3b-c}}{[T]^{c+d}}$$

$$\begin{cases} b+c=1 \\ c+d=2 \quad \Rightarrow \quad c=2-d\,; \quad b=1-c=1-2+d=d-1 \\ a+d-3b-c=1 \quad \Rightarrow \quad a=1-d+3b+c = 1-d+3d-3+2-d \\ \qquad = d \end{cases}$$

$$\Rightarrow F_{ar} \propto R^d \rho^{d-1} \eta^{2-d} v^d \Rightarrow F_{ar} = C R^d \rho^{d-1} \eta^{2-d} v^d.$$

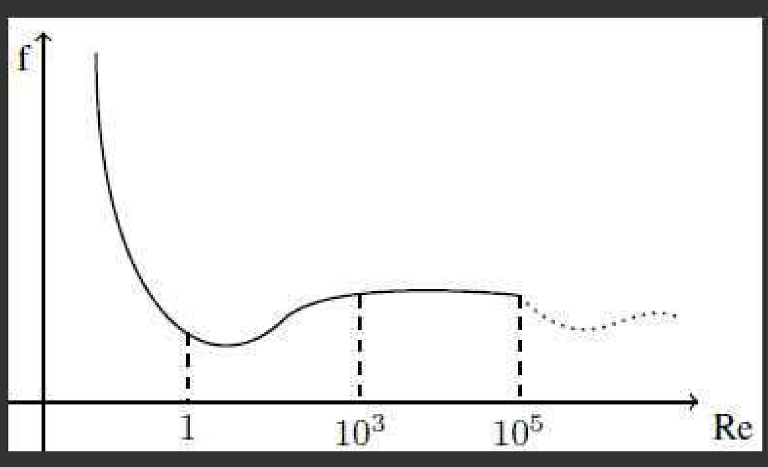
$$d=1 \Rightarrow F_{ar} = \underbrace{C_1 R \eta}_{b_1} v = b_1 v \rightarrow \text{Arrasto linear.}$$

$$d=2 \Rightarrow F_{ar} = \underbrace{C_2 R^2 \rho}_{b_2} v^2 = b_2 v^2 \rightarrow \text{Arrasto quadrático.}$$

ARRASTO LINEAR OU QUADRÁTICO?

$$F = R^2 \rho v^2 f\left(\frac{\rho R v}{\eta}\right) \quad ; \quad \frac{\rho R v}{\eta} = Re \rightarrow \text{N}^{\text{o}} \text{ de Reynolds.}$$

Curva empírica:



Regimes:

i) $Re < 1 \Rightarrow \frac{\rho R v}{\eta} < 1 \Rightarrow v < \frac{\eta}{\rho R}$ → Baixas velocidades.

$$f \simeq \frac{C_1}{Re} = \frac{C_1\,\eta}{\rho R v} \Rightarrow F_{ar} = \frac{C_1 \eta}{\rho R v} \cdot R^2 \rho v^2 = C_1 R \eta v.$$

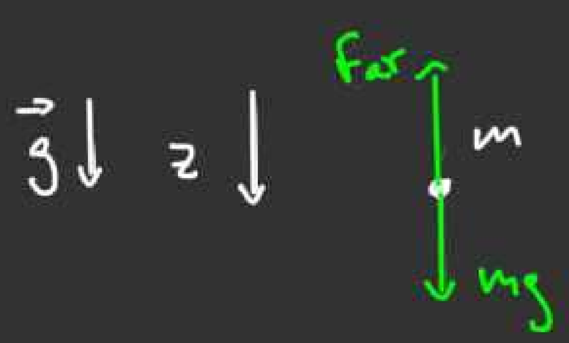
ii) $10^3 < Re < 10^5 \Rightarrow 10^3 < \frac{\rho R v}{\eta} < 10^5 \Rightarrow 10^3 \frac{\eta}{\rho R} < v < 10^5 \frac{\eta}{\rho R}$.

→ Altas velocidades.

$$\Rightarrow f \simeq C_2 \Rightarrow F_{ar} = C_2 R^2 \rho v^2 \rightarrow \text{Arrasto quadrático.}$$

Qual a velocidade terminal de uma gota de chuva típica?

Raio médio = 0,2 cm = $2 \cdot 10^{-3}$ m
Massa média = 0,034 g = $3{,}4 \cdot 10^{-5}$ kg
Dens. ar (20°C) = 1,2 kg/m³
Visc. ar (20°C) = $1{,}7\ 10^{-5}$ kg/m·s



$$m\frac{dv}{dt} = mg - F_{ar}$$

Vel. terminal : $\frac{dv}{dt} = 0 \Rightarrow mg = F_{ar}$.

$$Re = \frac{\rho R v}{\eta} \sim \frac{10^0 \cdot 10^{-3}}{10^{-5}} v = 10^2 \cdot v$$

Linear: $10^2 v < 1 \Rightarrow v \sim 10^{-2}$ (Péssimo)

Quadrático: $10^3 < 10^2 v < 10^5 \Rightarrow 10^1 < v < 10^3$ (Bom!)

$$mg = b_2 v_t^2 \Rightarrow v_t = \sqrt{\frac{mg}{b_2}} = \sqrt{\frac{mg}{C_2 R^2 \rho}}$$

Esfera : $C_2 \simeq 0{,}2$

$$v_t \simeq \sqrt{\frac{3{,}4 \cdot 10^{-5} \cdot 9{,}8}{0{,}2 \cdot (2 \cdot 10^{-3})^2 \cdot 1{,}2}} \simeq 18{,}6\ \frac{m}{s}.$$