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MOVIMENTO DE PROJÉTEIS COM ARRASTO

$$m\frac{d\vec v}{dt} = m\vec g - |F_{ar}|\hat v,$$

Ar: $Re \sim \frac{10^0 R v}{10^{-5}} = 10^5 R v$

Água: $Re \sim \frac{10^3 R v}{10^{-3}} = 10^6 R v$

$Re = \frac{\rho R v}{\eta}$; Óleo de mamona: $Re \sim \frac{10^3 R v}{10^0} = 10^3 R v$

$$v^2\hat v = |\vec v||\vec v|\hat v = |\vec v|\vec v = v\vec v$$

Linear: $Re \lesssim 1$; Quadrático: $10^3 \lesssim Re \lesssim 10^5$.

Quadrático: $m\frac{d\vec v}{dt} = m\vec g - b v^2\hat v \Rightarrow \frac{d\vec v}{dt} = \vec g - \frac{b}{m} v\vec v$

$$\begin{cases} \frac{dv_x}{dt} = -\frac{b}{m}\sqrt{v_x^2+v_z^2}\, v_x \\ \frac{dv_z}{dt} = -g - \frac{b}{m}\sqrt{v_x^2+v_z^2}\, v_z \end{cases} \Rightarrow \text{Acoplado!}$$

Lançamento do projétil no óleo de mamona! Arrasto linear.

$$m\frac{d\vec v}{dt} = m\vec g - b\vec v \Rightarrow \underbrace{\left(\frac{d\vec v}{dt} + \frac{b}{m}\vec v\right)e^{bt/m}}_{\frac{d}{dt}\left[\vec v(t)e^{bt/m}\right]} = \vec g\, e^{bt/m}$$

$$\frac{d}{dt}\left[\vec v(t)e^{bt/m}\right] = \vec g\, e^{bt/m}$$

$$\int_{\vec v(0)}^{\vec v(t)e^{bt/m}} d\left[\vec v(t)e^{bt/m}\right] = \int_0^t \vec g\, e^{bt/m}\,dt$$

$$\Rightarrow \vec v(t)e^{bt/m} = \vec v(0) + \vec g\frac{m}{b}\left(e^{bt/m} - 1\right).$$

$$\Rightarrow \vec v(t) = \vec v(0)e^{-bt/m} + \frac{m\vec g}{b}\left(1 - e^{-bt/m}\right).$$

$$v_x = v_{0,x}e^{-bt/m},$$

$$v_z = v_{0,z}e^{-bt/m} - \frac{mg}{b}\left(1 - e^{-bt/m}\right) \Rightarrow v_{z,t} \to -\frac{mg}{b}.$$

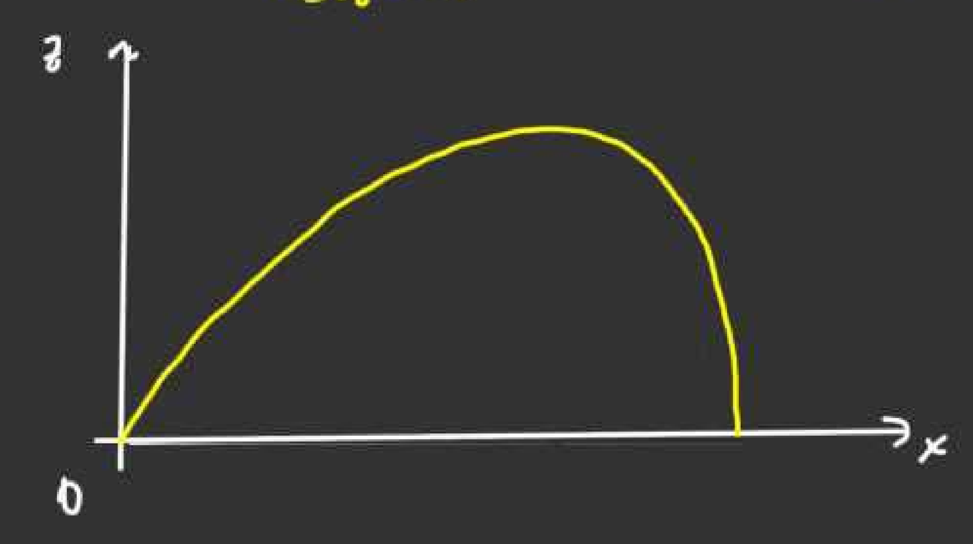
$$\frac{d\vec r}{dt} = \vec v = \frac{m\vec g}{b} + \left(\vec v(0) - \frac{m\vec g}{b}\right)e^{-bt/m}$$

$$\int_{\vec r(0)}^{\vec r(t)} d\vec r = \int_0^t \left[\frac{m\vec g}{b} + \left(\vec v(0) - \frac{m\vec g}{b}\right)e^{-bt/m}\right]dt$$

$$\vec r(t) = \frac{m\vec g}{b}t + \left(\frac{m\vec v(0)}{b} - \frac{m^2\vec g}{b^2}\right)\left(1 - e^{-bt/m}\right), \quad \vec r(0) = 0.$$

$$\Rightarrow \begin{cases} x = \frac{m v_{0,x}}{b}\left(1 - e^{-bt/m}\right), \\ z = -\frac{mgt}{b} + \left(\frac{m v_{0,z}}{b} + \frac{m^2 g}{b^2}\right)\left(1 - e^{-bt/m}\right) \end{cases}$$

$$z(x) = \left(\frac{mg}{b v_0\cos\theta} + \tan\theta\right)x + \frac{m^2 g}{b^2}\ln\left(1 - \overset{=y}{\frac{bx}{m v_0\cos\theta}}\right)$$



$$x(t\to\infty) \to \frac{m v_{0,x}}{b}.$$

$$\ln(1-y) = -y - \frac{y^2}{2} - \frac{y^3}{3} - \dots$$

$$z = \left(\cancel{\frac{mg}{b v_0\cos\theta}} + \tan\theta\right)x + \frac{m^2 g}{b^2}\left(\cancel{-\frac{bx}{m v_0\cos\theta}} - \frac{b^2x^2}{2m^2v_0^2\cos^2\theta} - \frac{b^3x^3}{3m^3v_0^3\cos^3\theta} - \dots\right)$$

$$= \tan\theta\, x - \frac{gx^2}{2v_0^2\cos^2\theta} - \frac{bgx^3}{3mv_0^3\cos^3\theta} + O(b^2)$$

x muito pequeno : $z \sim \tan\theta\, x - \frac{gx^2}{2v_0^2\cos^2\theta}$

$b \to 0$

Alcance

1ª ordem em b : $z = x\underbrace{\left[\tan\theta - \frac{gx}{2v_0^2\cos^2\theta} - \frac{bgx^2}{3mv_0^3\cos^3\theta}\right]}_{2\text{ soluções}} = 0$

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Alcance: $x = \frac{3mv_0\cos\theta}{4b}\left(\sqrt{1 + \frac{16 b v_0\operatorname{sen}\theta}{3mg}} - 1\right)$

Taylor: $\sqrt{1+\lambda} = 1 + \frac{\lambda}{2} - \frac{\lambda^2}{8} + \dots$

$\Rightarrow$ Alcance $x = \frac{v_0^2\operatorname{sen}(2\theta)}{g}\left(1 - \frac{4bv_0\operatorname{sen}\theta}{3mg}\right) + O(b^2)$