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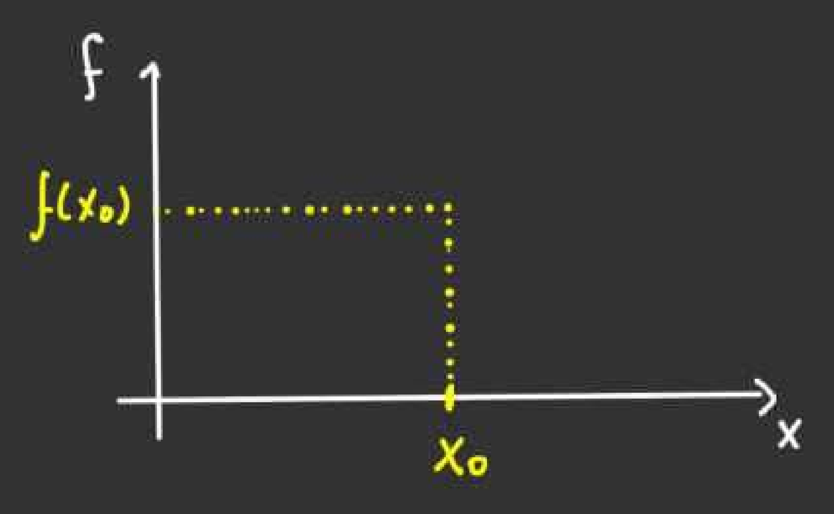
EXPANSÃO EM SÉRIE DE TAYLOR



Não conheço $f(x)$, $x \neq x_0$.
Conheço $f(x_0)$, $f^{(n)}(x_0)$.
Expansão em Taylor $\Rightarrow$ $f(x)$, em torno de x_0.

$$f(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)^2 + a_3(x-x_0)^3 + \ldots = \sum_{n=0}^{\infty} a_n (x-x_0)^n.$$

$$f(x_0) = a_0$$

$$f'(x) = a_1 + 2a_2(x-x_0) + 3a_3(x-x_0)^2 + 4a_4(x-x_0)^3 + \ldots$$

$$f'(x_0) = a_1$$

$$f''(x) = 2a_2 + 3\cdot 2\, a_3(x-x_0) + 4\cdot 3\, a_4 (x-x_0)^2 + \ldots$$

$$f''(x_0) = 2a_2 \Rightarrow a_2 = \frac{1}{2} f''(x_0).$$

$$f'''(x_0) = 3\cdot 2\cdot 1\cdot a_3 \Rightarrow a_3 = \frac{1}{3!} f'''(x_0)$$

$$a_n = \frac{1}{n!} f^{(n)}(x_0)$$

$$\Rightarrow f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

Exp. Taylor em torno de x_0

Exemplo: $f = \frac{1}{1+x}$, em torno de $x_0 = 0$.

$f(0) = 1$,

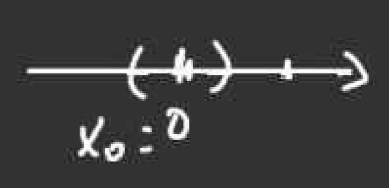
$f'(0) = -\frac{1}{(1+x)^2}\Big|_0 = -1$,

$f''(0) = +\frac{2}{(1+x)^3}\Big|_0 = +2$,

$f^{(3)}(0) = -\frac{3\cdot 2}{(1+x)^4}\Big|_0 = -3!$

$f^{(4)}(0) = +4!$

$f^{(n)}(0) = (-1)^n\, n!$



$$f(x) = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + x^4 - \ldots$$

$f(0{,}5) = ?$ Exato: $f(1/2) = \frac{1}{1+1/2} = \frac{2}{3} = 0{,}6666\ldots$

Expansão: $f(1/2) = 1 - \frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \ldots$

Sol em ordem 0: $f(1/2) = 1$.

" " " 1: $f(1/2) = \frac{1}{2}$.

" " " 2: $f(1/2) = 1 - 0{,}5 + 0{,}25 = 0{,}75$

" " " 3: $f(1/2) = 1 - 0{,}5 + 0{,}25 - 0{,}125 = 0{,}625$

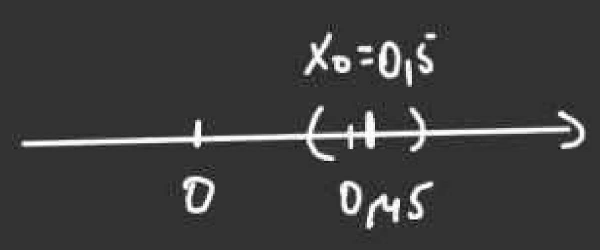
$f(0{,}5) = ?$ Expandir $f = \frac{1}{1+x}$ em torno $x_0 = 0{,}45$

$$f(x) = \frac{1}{1+0{,}45} - \frac{1}{(1+x)^2}\Big|_{0{,}45}(x-0{,}45) + \ldots$$

$$= \frac{1}{1+0{,}45} - \frac{1}{(1+0{,}45)^2}(x-0{,}45) + O(x^2)$$

Sol. ordem 0: $f(1/2) = \frac{1}{1+0{,}45} \approx 0{,}69$

" " 1: $f(1/2) = \frac{1}{1+0{,}45} - \frac{(0{,}5-0{,}45)}{(1+0{,}45)^2} \approx 0{,}665$



Exemplos: expansões em torno de 0:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

$$\operatorname{sen} x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

Convergem $\forall\, x$.

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

$$e^{i\theta} = \cos\theta + i \operatorname{sen}\theta$$