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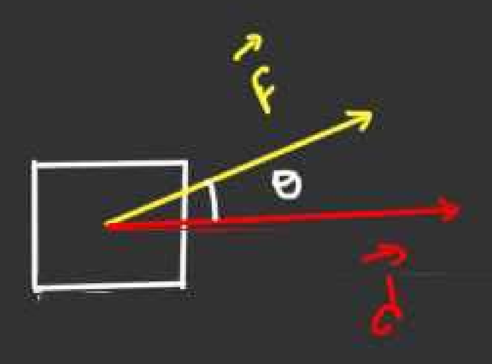
INTEGRAIS DE TRABALHO

Caso particular: Força constante e trajetória retilínea



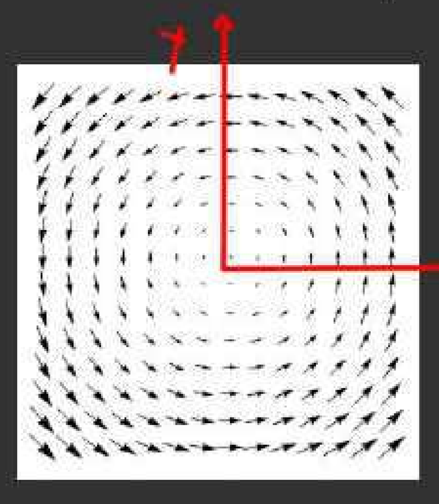
$$W = \vec{F} \cdot \vec{d} = F\, d \cos\theta \, .$$

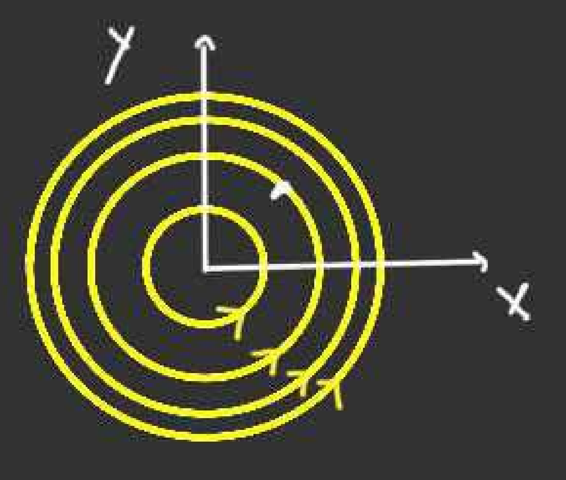
CAMPO DE FORÇA

Independente do tempo: $\vec{F} : \mathbb{R}^d \to \mathbb{R}^d$
$\vec{r} \mapsto \vec{F}(\vec{r})\, .$

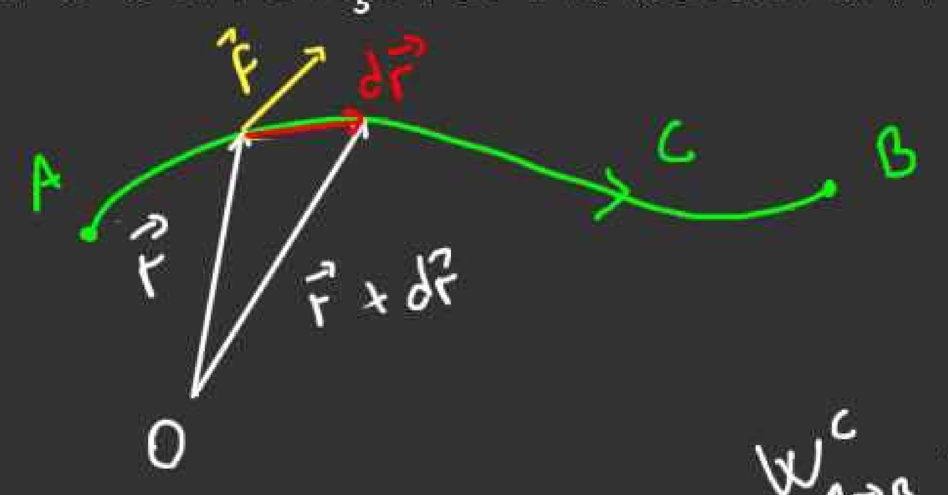
Dependente do tempo: $\vec{F} : \mathbb{R}^d \times \mathbb{R} \to \mathbb{R}^d$
$(\vec{r}, t) \mapsto \vec{F}(\vec{r}, t)\, , \quad \frac{\partial \vec{F}}{\partial t} \neq 0 \, .$

Exemplo: $\vec{F}(x,y) = -y\hat{\imath} + x\hat{\jmath}\, .$





CAMPO DE FORÇA E TRAJETÓRIA ARBITRÁRIOS



$\vec{F}$ ind. do tempo

$$dW = \vec{F} \cdot d\vec{r}$$

$$\boxed{W^C_{A \to B} = \int_{A,C}^{B} \vec{F} \cdot d\vec{r}}$$

$\vec{F}$ depend. do tempo:

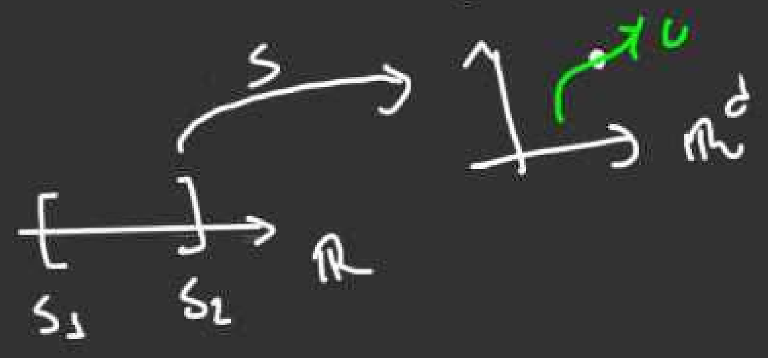
$\vec{v} = \frac{d\vec{r}}{dt}$

$$\boxed{W^C_{A \to B} = \int_{A,C}^{B} \vec{F}(\vec{r}, t) \cdot \vec{v}(t)\, dt}$$

Note, em geral:

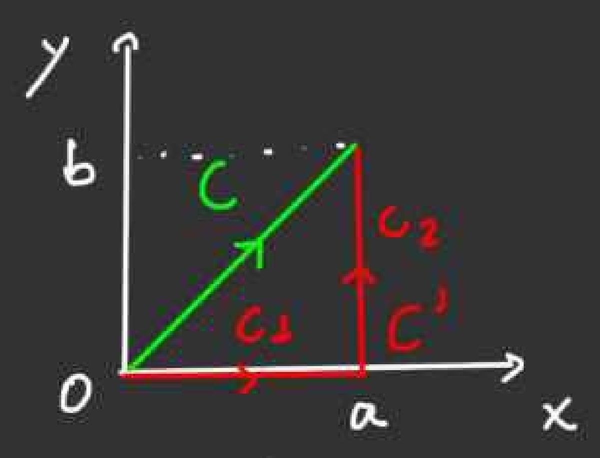
$$W^C_{B \to A} = \int_{B,C}^{A} \vec{F} \cdot d\vec{r} = -\int_{A,C}^{B} \vec{F} \cdot d\vec{r} = -W^C_{A \to B}\, .$$

Parametrização s: $\vec{r}(s_1) = \vec{r}_A\, ; \ \vec{r}(s_2) = \vec{r}_B\, ; \ \vec{r} = \vec{r}(s)\, .$



$$\boxed{W^C_{A \to B} = \int_{A,C}^{B} \vec{F}(\vec{r}(s)) \cdot \frac{d\vec{r}}{ds}\, ds}$$

Exemplo: Trabalho de $\vec{F}(x,y) = xy\hat{\imath} + cx\hat{\jmath}$, ao longo de C e C'.



$y = \frac{b}{a} x \to$ para C.

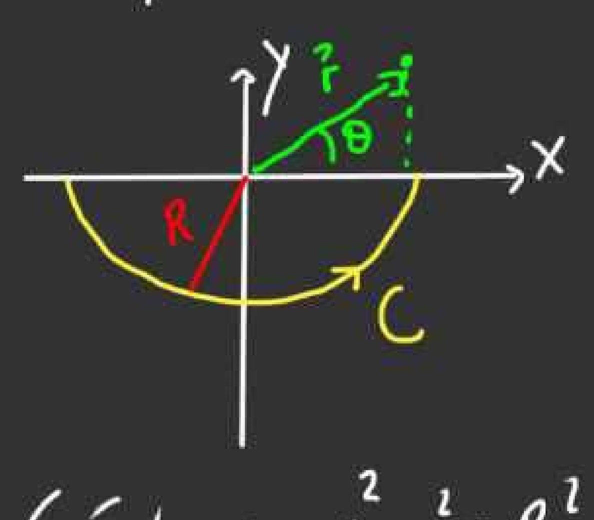
$$W^C = \int_C \vec{F} \cdot d\vec{r} = \int_C (F_x\, dx + F_y\, dy) = \int_C xy\, dx + c\int_C x\, dy$$

$$= \int_0^a x \cdot \frac{b}{a} x\, dx + c\int_0^b \frac{a}{b} y\, dy = \frac{b}{a}\frac{a^3}{3} + \frac{ac}{b}\frac{b^2}{2} = ab\left(\frac{a}{3} + \frac{c}{2}\right)$$

$$W^{C'} = \int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r} = \int_{C_1} (F_x\, dx + F_y\, \overset{0}{dy}) + \int_{C_2} (F_x\, \overset{0}{dx} + F_y\, dy)$$

$$= \int_{C_1} x \overset{0}{y}\, dx + \int_{C_2} c \overset{a}{x}\, dy = \int_0^b c\, a\, dy = abc\, .$$

Exemplo: Trabalho de $\vec{F}(x,y) = ay\hat{\imath} + b\hat{\jmath}$ ao longo do semicírculo C.



$$W^C = \int_C \vec{F} \cdot d\vec{r} = \int_C (F_x\, dx + F_y\, dy)$$

$$= \int_C ay\, dx + \cancel{\int_C b\, dy}^{\,0} = a\int_C y\, dx$$

Círculo: $x^2 + y^2 = R^2$
$\Rightarrow y = \pm\sqrt{R^2 - x^2}$

$$= -a\int_{-R}^{R} \sqrt{R^2 - x^2}\, dx \; ; \quad x = R\, \text{sen}\,\theta\, .$$

...

Sol. 2: θ (coord. polares)

$$\vec{r} = R(\cos\theta\, \hat{\imath} + \text{sen}\,\theta\, \hat{\jmath})\, , \quad \theta \in [\pi, 2\pi]$$

$$\vec{F} = aR\, \text{sen}\,\theta\, \hat{\imath} + b\hat{\jmath}\, .$$

$$W^C = \int_\pi^{2\pi} (aR\, \text{sen}\,\theta\, \hat{\imath} + b\hat{\jmath}) \cdot R(-\text{sen}\,\theta\, \hat{\imath} + \cos\theta\, \hat{\jmath})\, d\theta$$

$$= \int_\pi^{2\pi} (-aR^2\, \text{sen}^2\theta\, d\theta + bR\cos\theta\, d\theta)$$

$$= -aR^2 \int_\pi^{2\pi} \frac{1 - \cos 2\theta}{2}\, d\theta + \cancel{bR\int_\pi^{2\pi} \cos\theta\, d\theta}^{\,0} = -\frac{aR^2}{2}\left(\theta - \frac{\text{sen}\, 2\theta}{2}\right)_\pi^{2\pi}$$

$$= -\frac{aR^2\pi}{2}\, .$$